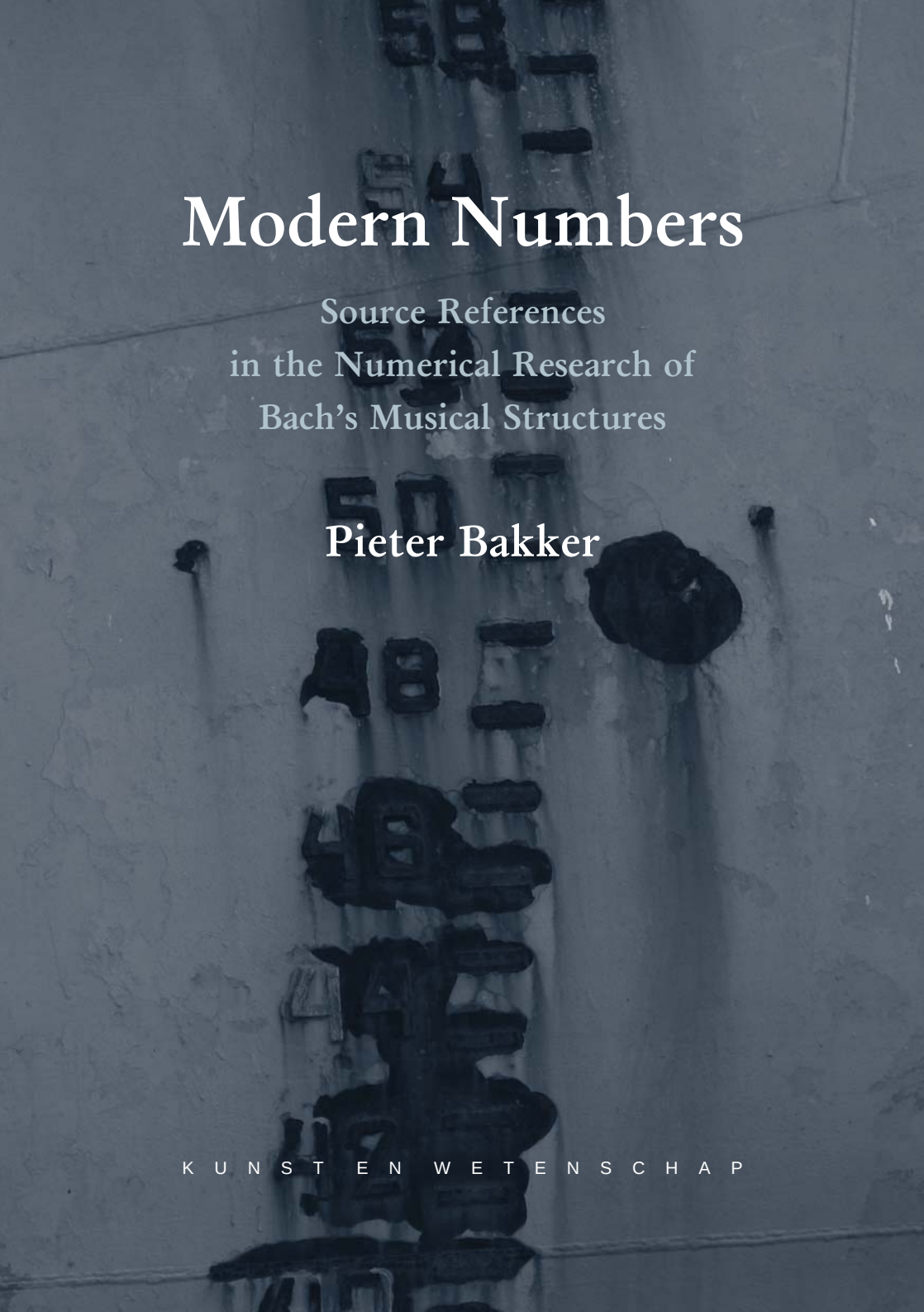
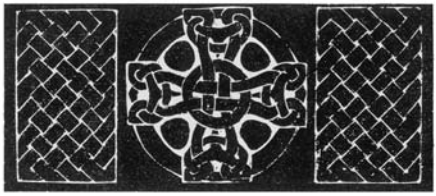


Modern Numbers

Source References in the Numerical Research of Bach's Musical Structures

Pieter Bakker

KUNST EN WETENSCHAP



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Eighteen years ago, an article I wrote about the seventeenth-century organist and music-theorist Andreas Werckmeister appeared in the Dutch Tijdschrift voor Muziektheorie. *My main intent at that time was to determine which sources this theorist, who was prototypical for his time and region, had used. In music history, Werckmeister had been regarded since the 1950s as someone who, in many ways, was 'medieval' and had opposed the emerging Enlightenment. My conclusion was that he was a clear representative of the aftereffects of humanism, who in fact resisted scholastic ideas sometimes, but who, on the other hand, was hardly touched by the new era, the way it is represented by Johann Mattheson, for example.*

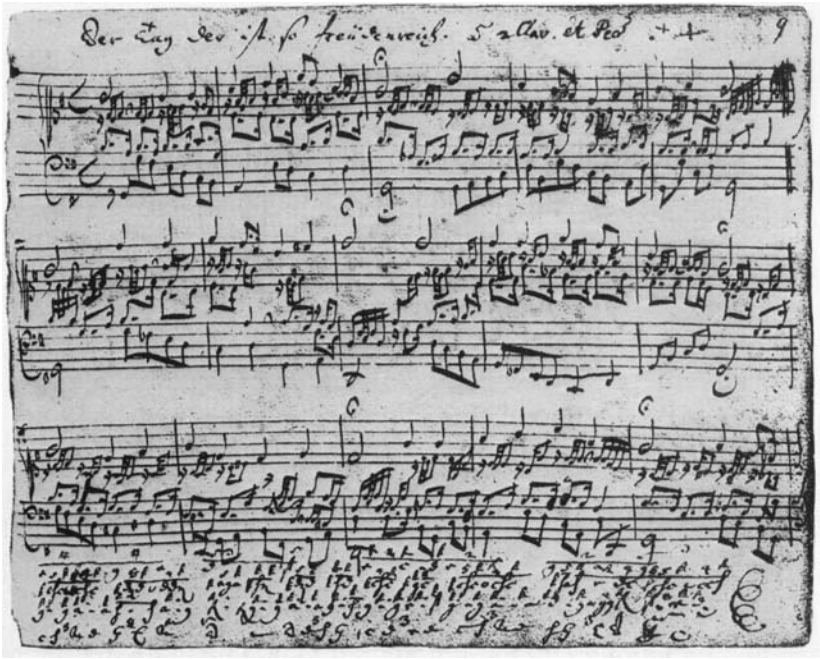
By mapping out Werckmeister's sources one gets a clear picture of his position in music history, which isn't unimportant in light of the fact that since the 1950s he has increasingly been used as a source himself. The symbolically interpreted numbers in his writing are however regularly taken out of their original context of intervals and harmony and used, among other things, for analyses of form, that is, for the ratio of lengths or durations in Bach's works. After noting this and initially being surprised about the confusion, the phenomenon itself started to interest me.

The question presents itself where the starting point lies of this modern analytical research, which, on historical grounds, connects objective formal analysis with symbolic interpretation, and further what its original inspiration has been and what it says about the historical position of the music-theoretical research in the second part of the last century. The study that follows is not in the first place about Werckmeister and his music-theoretical contemporaries, but is rather a short history of the use of historical and biographical sources in, what I call, the numerical research of Bach's musical form – that is, the research about numbers in connection with formal structure – as it has manifested itself since the 1920s.

Expositions about symbolic numbers have a tendency to become endless, which is something I want to avoid here. What to one person seems a fact, may seem no more than a coincidence to someone else. That's why I have chosen to concentrate on the historical and biographical source references that are being used in the research into structural numbers in Bach's work and not on the analyses themselves. The footnotes take up quite a

lot of space and come with a wealth of quotations. The intention is that the ideas and positions and also the argumentation, the local colour, become clear to those who don't have immediate access to the literature.

Pieter Bakker *Schraard, February 2015*



Prelude to the chorale *Der Tag ist so freudenreich* from Johann Sebastian Bach's *Orgelbüchlein* with organ notation in the margin at the bottom

The interest in Johann Sebastian Bach, which was already great during his lifetime, has, since his death in 1750, never abated. This is also true of the historical research into his life and work. Yet it wasn't until a hundred and seventy years after his death that something was written about the numbers and groups of numbers that some discerned in his compositions. Since the 1920s, ever more studies about numbers as a form-determining factor have been published. Then, at the end of the 1930s, those numbers were also given symbolic meaning. The question is whether in the music-theoretical and biographical source material that has been put forward from that time, alongside score analyses, a connection between proportion and symbolic interpretation can be found in Bach's work. For in much music-theoretical literature a connection between numbers and symbols and the formal structure of Bach's work is now being based on historical grounds, while primary sources about his work methods on this point are so far lacking.

The numerical research into Bach's musical form centres mainly, but not solely, on symbolic numbers. The concept, as it is being used here, therefore covers all formal analyses where numbers, found or applied in music written by Bach, are the starting point of the research. The first attempt of such a numerical project was made in the 1920s by an analytical study of the structural numbers in the work of Johann Sebastian Bach (1685–1750), but as yet there was no mention of a symbolic meaning. Symbolism in Bach *was* being studied in those days, but only in later studies, from the 1930s on, was a connection with structural numbers often made.
A direct connection between these symbolic structural numbers and a music-theoretical source was lacking initially, but in the 1950s researchers thought they had found it in the writings of Andreas Werckmeister (1645–1706). This seventeenth-century author had written extensively, in the Neoplatonic sense, but also more allegorically, about the symbolic interpretation of numbers.[1] All numbers, in Werckmeister, go back to the one or the Unity. The triunisonus 1 : 2 : 3, in musical notes C-c-g, is perfectly consonant and depicts the Trinity.[2] The major triad with the ratio 4 : 5 : 6, that is built on the third octave, in musical notes c-e-g, shows the mirror image of the Trinity, since one hears this trisonus as unitrisonus.[3] These numbers form the speculative basis of Werckmeister's theory. The image of the triad comes originally from Johannes Lippius (1585–1612).[4]

A less fundamental and different kind of example of Werckmeister's number symbolism is the interpretation of the ratio 5 : 6, where the animal number six, as related to the preceding number five, which in an earlier context is referred to as the human number, forms the smallest superparticular ratio within the harmonic numbers. The sad character of these minor third ratio shows that the animal nature is small and slight.[5]
The ratios are derived from the numero senario of Gioseffo Zarlino (1517–1590), to which Sethus Calvisius (1556–1615), Lippius and, in imitation of them later, Werckmeister, have added the eight.[6] These seven numbers relate in their original context to matters of pitch, but were applied more broadly by researchers in the twentieth century.[7] Werckmeister's interpretation of numbers concerns however *musica theoretica,* the speculative music, and not *musica poetica,* which concerns itself with the writing of music, with its lengths.[8] Still Werckmeister does connect the speculative music to actual practice in the building of chords and in voice-leading.[9] The crux to him is always in how close the numbers approach unity or equality.[10] But nowhere can you read that the harmonic numbers can also be seen as structural numbers. This cannot be derived from Werckmeister's writings, or, so far, from other music-theoretical sources of his contemporaries.

Pioneer

It was the cantor and musicologist Wilhelm Werker (1873–1948) who, in the early 1920s, first published something about numbers in Bach's work. In his *Studien über die Symmetrie im Bau der Fugen und die motivische Zusammengehörigheid der Präludien und Fugen des 'Wohltemperierten Klaviers' von Johann Sebastian Bach* he talks about the formal function of numbers in figures and ratios within the composition. Werker did not care for the kind of number interpretation that would later play a role. Yet Arnold Schering would, strangely enough, criticize him by saying that number symbolism was his demon.[11] It may be more likely that Werker's successors in the 1930s wanted to exorcise this supposed demon with Christian symbols.[12]
To Werker, as he remarks in his book, Bach is the brilliant and diligent mathematician, never the lazy or even mystical speculator in numbers.[13] He refers in this context to an item by Spitta from Forkel's biography about Bach's insight into room acoustics.[14] Werker is mainly concerned

with the building plan of Bach's compositions. He fulminates against an overvaluation of feeling in music criticism.[15] In his vision Bach had not only a big heart but also a clever and visionary insight into the art.[16] In many places in his book Werker calls Bach the architect of his works. He advises us to regard the work of the few masters, who were forced to create, as products of nature, so we will realize how necessary their formal structures were.[17] A certain spirituality can be discerned in those words, as in the description of his symmetric structures, though the author does not elaborate on it.

It is no coincidence that Arnold Schönberg (1874–1951) welcomed Werker's study.[18] Nor is the sharp attack by Schering a coincidence, as he reproaches Werker for positioning the germ for a composition in abstract thought and not having the need to ground his theory more deeply, that is, more deeply in the psyche.[19] In his rejection of the intrusive accent on feeling in the preceding period and his strong emphasis on the formal aspect of music, Werker was a typical representative of modernity.[20]

Werker's approach is exclusively analytical. Arguments, from historical sources, are rarely given. Werker even assumes that Bach's sons had no idea of the architecture of their father's work, as otherwise those forms would not have crumbled in their hands.[21] A direct link between music-theoretical or biographical sources and Bach's use of numbers is lacking save for one curious exception. Werker recommends studying the 'strangely mysterious algebraic formulas' under the prelude to the chorale *Der Tag, der ist so freudenreich,* since they would reveal something about Bach's work method.[22] However, it is obvious from the manuscript that they do not concern ratio or structure but are rather a notation in organ tablature. Yet, this explanation Werker pushes aside as unlikely. That Werker puts more weight in analysis than in an underpinning from sources is apparent from the preface and many passages in his book.[23]

Rise

According to Wilibald Gurlitt (1889–1963) Werker's ideas were very controversial during his lifetime.[24] More successful in music-theoretical circles were the just as strictly formal ideas of Wolfgang Graeser (1906–1928), even though he wasn't wholeheartedly positive in regards to the state of affairs in musicology.[25] That his authoritative publication of Bach's

Kunst der Fuge immediately found its way into music practice must have added to his popularity, as well as the metaphysical and psychological foundation of his research inspired by Oswald Spengler.[26] Among prominent musicologists, however, there was, besides admiration, also strong criticism of his work.[27]

The publications about the connection between numbers and form in Bach from the generation after Werker were less strictly mathematical and more exegetic in nature. Especially the work of the theologian Friedrich Smend (1893–1980) found immediate acceptance. An important difference with Werker is that Smend interprets the numbers he has arrived at symbolically. The numbers can refer to quantities or symbolic numbers in the Bible.[28] On top of that Smend finds numbers by applying gematria, using a number alphabet that he discovered in the work of the poet Picander, a pseudonym for Christian Friedrich Henrici (1700–1764).[29] The gematrian technique he got from the Dutch pianist Henk Dieben (1902–1956), without, however, acknowledging him in his publications.[30]

With Smend the interpretation of numbers and their application are in fact more important than the formal structure itself. Much more than Werker he pays attention to historical sources, albeit solely with regard to the symbolism of the numbers. Still he is not able to make a direct connection between music-theoretical or biographical writings from the time and the results of his analyses. Nor was Dieben able to make such a connection. But he points to a fact others had mentioned before: that Bach himself had probably made a remark alluding to the possibility of melodically using the letters in the name of Bach as notes.[31] Especially in the first of his not very extensive articles Dieben is still very close to the modern spirit of Werker.

Smend is enthusiastic about his acquaintance with Dieben, but does think he discerns a lack of theological knowledge in him and therefore concludes that apart from taking something from the relationship he is also giving something and is therefore justified in not mentioning Dieben as a source.[32] In an article Smend distances himself from Werker by saying that he is only right in the most general sense of the word. Moreover, he finds Werker's tone arrogant and repulsive.[33] Werker's pioneering spirit is thus played down by an unrelated argument. In the words of the Dutch writer Multatuli: 'To such a poet one should bow one's head but... he beats his wife!' A conclusion which relieves one of 'the tribute that is due'.

The teacher and musician Martin Jansen (1885–1944) was in close contact with Smend. He accords Werker more or less the honour he deserves, but is still very sparing in his appreciation. Jansen, according to himself, reopened the research into numbers in Bach's work, after Werker had obstructed the road to further research by his rash and imprudent initiative.[34] Just as in Smend, there is a lot of theological interpretation in Jansen. He presents no music-theoretical or biographical sources from Bach's time. But he does try to put Bach's work into a larger historical framework. The thinking of earlier generations, in which numbers are not just quantitative but part of an ordering principle, dates in his vision from the Middle Ages.[35]

The music theory of Andreas Werckmeister, that many researchers have linked to Bach, has also often been described as medieval or scholastic, just like the 'past generations' Jansen speaks of.[36] At least one author went so far in this that according to him Werckmeister didn't just reject the Enlightenment but also the aftereffects of humanism.[37] From Werckmeister's arguments and consulted sources it is however clear that he himself was a representative of the lingering humanism and that he rejected scholasticism which at the beginning of the seventeenth century was making a resurgence at Lutheran universities.[38] He was on the other hand hardly touched by the new age, the way it is, for example, represented by the enlightened Mattheson.

Dissemination

In the generation after Smend and Jansen, counting gradually became commonplace. On top of that, number theories with their symbolic interpretations became part of musical practice. The Dutch harpsichordist and conductor Hans Brandts Buys (1905–1959) is a good example of this development. For the counting of small entities, he continued on the work of Werker and Smend, but added the counting of the entities in measured time.[39] As a new approach he introduced a description of the cross-shaped construction of Bach's St. Matthew Passion.[40]

The historic arguments in Brandts Buys' study are few and not especially strong. Bach rarely wrote music without a symbolic intention, he contends. A remark in a *Generalbasslehre* from 1738, attributed to Bach, of which the authenticity is problematic, namely that a hopeless noise will

ensue when the goal of music, which is to honour God and to soothe man's inner nature, isn't paid attention to, was as far as he was concerned enough to reach the conclusion that 'the recognition of symbols has to be the basis of every interpretation'.[41] That is a quite a big leap in his argument. Yet it is interesting to mention Brandt Buys here, since he, as a practicing musician, places the application of numbers and symbols on formal structure in the context of historical performance practice.[42]

In this whole complex of numbers, symbols and performance we are starting to see the musicology of the 1970s, and the aftereffects in our own time. The connection of numbers with formal analysis in Bach's work, especially the counting of bars, became more and more an accepted part of musicology. In Germany the development is the same as in the Netherlands. The generalists have also adopted number research in their work. Without the explicitly formal aspect and no emphasis on symbolic numbers, the combination of symbol and performance practice was, for that matter, already present in Arnold Schering's work in the 1920s.[43]

The linking of number symbolism and form has since the 1950s been an integral part of musicology. An extensive historical argument in this field we find in Fritz Feldmann (1905–1984). Feldmann dates the counting of musical entities as starting in the Middle Ages and sees a link between the quadrivium and the trivium which the humanists apparently emphasized.[44] It seemed logical to Feldman to also apply counting to the figurations. As a source that would point in that direction, he mentions a passage in Tinctoris. An unfortunate reference, for that passage deals exclusively with the treatment of dissonance in counterpoint.[45] In addition, one has reason to think that with the humanists the quadrivium would be emphasized. So it is mostly to quadrivial theoretic music of especially seventeenth-century sources to which, in connection with numbers, twentieth-century musicologists refer. What is striking, is that Feldman, for the counting of measures and in order to underline the universality of number symbolism, does not take Bach as an example but Georg Friedrich Händel (1685–1759).[46]

Generalists like Walter Blankenberg (1903–1986) and especially Rolf Dammann (1929–2012) also make room for numbers and counting in connection with musical architecture and its symbolic and historic interpretation. In both the counting of bars is part of their symbolic interpretations.[47] What, in retrospect, makes Dammann's research less solid is the

fact that he interpreted the symbolic numbers from music-theoretical sources like Lippius and Werckmeister, where they were originally presented in connection with harmony, without further source references and much more broadly. The connection with formal structure was made by Dammann himself. Thus he uses the word *Musikbau,* that he found in Werckmeister, also for the architecture of a composition, while in the source it is only applied to the ratio of tones.[48] The projection of the numbers Gioseffo Zarlino dirived from the numero senario linked to a seventeenth-century, symbolic interpretation, often adopted from Werckmeister, on the formal structure, has found a lot of followers right up to our own time.[49]

Influence

The generation after Blankenberg and Dammann produced a lot of books and articles about numbers in Bach's work. In the Netherlands, after Brandt Buys, who started it, especially choir conductors and organists did a lot of writing on this subject. Their scholarly activity developed from their musical practice. The advantage of such a background is obviously that a connection is made between the direct artistic contact with the work and scholarly research. Researchers generally have plenty of historical arguments at their disposal, but quite often show a lack of methodological insight when dealing with analysis and a limited critical capacity in the use of sources. But again, as with the generations before them, no direct connection between Bach's use of numbers in the musical form and music-theoretical writings of his time or any biographical data could be made.
In *Bach en het getal* Kees van Houten (1940) and Marinus Kasbergen (1936) build on the findings of Dieben and Smend and their application of the number alphabet.[50] They connect the interpretation of the numbers one to six, from the numero senario, plus the eight Werckmeister has added to these six, just like Dammann to the formal structure.[51] The fact that Werckmeister himself only talks about proportions of pitches is ignored. From Leibnitz' description of music as a hidden arithmetic practice by people who count unconsciously, the authors draw the conclusion that 'a symbolic use of numbers and so of proportions can be an aspect of this unconscious counting'. A further argument for this 'so' is lacking. The final proof for the link between numbers and ratios inside the form, Van

Houten and Kasbergen find in their analysis. 'All these examples seem so convincing to us that one more and more realizes that it can't be a question of coincidence.'[52] But that it can't be a question of coincidence is exactly what has to be proven. It is an argument that keeps returning and one we already find in Wilhelm Werker.[53]

Thijs Kramer (1938), in his *Zahlenfiguren im Werk Johann Sebastian Bachs,* also builds on Dieben's research even though he is a fervent opponent of Van Houten and Kasbergen. As far as the historical proof is concerned, Kramer limits himself mainly to the question whether someone in Bach's circle would have known something about his use of numbers in his compositions. He comes up with five arguments.[54]

In the first place, Bach's son Carl Philipp Emanuel has used the tone sequence b-a-c-h in transposed form in the fourteenth and forty-first bar of his Fantasie in C minor.[55] Fourteen is after all the sum of the letter values the name Bach adds up to. In the second place, there is the article in Mitzler's *Musicalische Bibiliothek,* written by, among others, Carl Philipp Emanuel, on the occasion of his father's death, that we find on page 158, followed by a poem reportedly composed of 104 lines.[56] So again we have numbers referring to Bach's name. In the third place, in 1747 Bach became the fourteenth member of Mizler's *Sozietät der musikalischen Wissenschaften.*[57] Again Bach's name. In the fourth place, there is the inscription in the glass goblet that Bach received from a pupil.[58] Again the number fourteen is hidden there in different ways. In the fifth place, Kramer draws our attention to the fact that Johann Nikolaus Forkel spoke of 'the Johann Sebastian Bach who is so proficient in mathematics'. Unfortunately, for this pronouncement he had to refer to a study by Heinrich Edelhoff, as the reading in question by Forkel was lost during the Second World War.[59] In the book, however, in the paraphrasing of Forkel's text a connection with temperament is made and not with formal structure. In tuning Bach is supposed to have acted according to nature instead of to calculations. Besides, Forkel wasn't a contemporary of Bach.

In *Der dritte Teil der Clavierübung von Johann Sebastian Bach,* Albert Clement (1962) is convinced of the importance of numbers in the architecture of Bach's compositions, referring again to Werckmeister and Leibniz. Just like Kramer, Clement brings up the 'usual' division according to the golden ratio.[60] In sources we hardly come across this division before the nineteenth century and it only acquires the special meaning some people

now attribute to it in the middle of that century.[61] Within the totality of the musical order there was, after all, no room for an irrational number in music theory.[62] For the beauty of musical proportions in the architecture, for example, he does refer to Vitruvius, but here too it is solely a question of rational numbers.[63] And just like Van Houten en Kasbergen, Clement uses the argument that the presence of a certain number 'can hardly be a coincidence'.[64] Shouldn't it be that in a scholarly work about a musical form one would have to show that there *is* no question of a coincidence.[65] By reversing the burden of proof an eventual opponent will later have to do the work himself.

Another example of a contemporary Dutch study in which the connection between number symbolism and formal structure is made, is *Jesu, meine Freude* by Arie Eikelboom (1948). Besides using interval proportions for illustration and commentary, Bach, with the help of gematria, is supposed to have used numbers for, among other things, bars. For his interpretation of numbers Eikelboom has especially consulted Werckmeister.[66] His analysis is, again, not supported by historical proof.

There are many examples of authors who have occupied themselves with numbers in Bach's work. A direct proof from a biographical source is, however, always lacking. No music-theoretical source from Bach's time applying number interpretation to formal structure has yet been found. Apart from the lack of number interpretation, it must also be said that, in general, writing about the number of bars and form only began in the second half of the eighteenth century. In 1752 the first part of *Anfangsgründe der musikalischen Setzkunst* by Joseph Riepel was published. Here the subject is the number of measures, but there is no symbolical interpretation. The subtitle, *Nicht zwar nach alt-mathematischer Einbildungs-Art der Zirkel-Harmonisten, sondern durchgehends mit sichtbaren Exempeln abgefasset,* already points in that direction, although this is again no reference to formal structure but to the interval theory and to temperament.[67]

New directions

The English musicologist Ruth Tatlow (1956) tries, starting in the early 1990s, to change directions. She is critical of the musicology of the decades since the Second World War, especially of the numerological specula-

tions.[68] Her own Bach research is based on the analyses of scores, but at the same time she makes a fervent plea for historical musicology. In the course of her research she designed a theory called proportional parallelism. In Bach's work she discovered the proportions 1 : 2 and 2 : 3 in the number of bars on different constructional levels.[69] In an earlier project, to her surprise, she thinks to have discovered that Bach applied a number alphabet to the structural form of a great number of cantatas, thus calculating number values for important lines in the text and applying the values he had found to determine the number of bars.[70] To her surprise, as she had earlier been critical in regard to Smend's research.

The statistical interpretations of the data from both Tatlow's research projects have so far been only partly worked out, but in the promised upcoming publication this will possibly be remedied. So far the project still shows other inadequacies. Proportional parallelism is now described as a concept, then as a method, a technique or a quality of Bach's work and later even as a theory.[72] One actually asks oneself the question: what is or runs parallel. Tatlow puts great store in a historical starting point, but it is exactly on this point that her own research is the most lacking. It is not always clear if and when she works deductively or inductively and whether she is making up or testing a theory.

Tatlow's research takes a strange turn when she, with the help of historical sources, arrives at her own symbolic interpretation of number proportions according to her method or theory of proportional parallelism.[74] The numbers are about the perfect proportions of the intervals. So by using these proportions Bach must have wanted to leave an *opus perfectum et absolutum*. A surprising conclusion, since it has up till now been thought that the concept of a piece of music as a finished and transferable work only originated in the nineteenth century.[75] Tatlow takes several steps to still come out at that conclusion.[76]

In the first place she brings up the rumours that were supposed to have circulated about Bach's working methods, but the sources she mentions do not confirm her assumption.[77] Secondly, in connection with the counting of bars, Tatlow points to the numbers of Andreas Werckmeister and others and the concept of proportions in Johann Mattheson. But Werckmeister only talks about intervals in relation to numbers and Mattheson certainly does not have exact arithmetic entities in mind when he speaks of proportions.[78] Her interpretation of the French word *harmo-*

nie in one of Bach's letters is supposed to take care of the connection between her arguments and a statement in a primary source.[79] In the third place the propensity for good proportions in combination with the fact that the counting of bars can already be found in Michael Praetorius leads to the conclusion that Bach applied certain proportions to the number of bars in his compositions.[80] In Praetorius it is however a question of adapting a piece of music to the length of the church service. To top everything off, Tatlow presents the well-known discussion between Mattheson and Johann Heinrich Buttstett about music in heaven. According to her this is about the continued existence of compositions in eternity, while in the sources there is only talk of the theoretical basis of music which would continue.[81] Tatlow's supposition that they are speaking about the harmony of proportions in a complete and transferable work is without any foundation.
Despite the shortcomings in the historical argumentation and the poor methodological set-up of her research, the great advantage of Tatlow's empirical approach of scores, compared to the research of many other musicologists in this field, is that she can come to a research result that is falsifiable. Her historical construction is however a house of cards.[82]

Modernity

During the last nearly one hundred years much has been written about numbers in Bach's work. Part of this musicological work has come up with a link between mostly symbolically interpreted numbers and the formal structure of Bach's compositions. But so far there is no reliable statistical interpretation of the analytical research data. Furthermore, no musicologist has yet been able to find a direct connection between contemporary music-theoretical or biographical sources and Bach's use of numbers. Bach could have known someone who may have meant a certain thing, he could have read a certain book about something that had perhaps something to do with it, he could have meant something without saying it explicitly, and so on.[83] Neither the historical hermeneutics of Dammann nor the empirical approach of Tatlow, to mention two very different examples from musicology, can obscure the fact that in reality nothing has been found in this area. This does not disqualify the research in question, but such an assessment puts the certainty with which these conclusions are

often reached and the stubbornly defended positions into perspective.
In the hundred and seventy years that preceded Werker, nothing was written about a connection between numbers or number symbolism and structural form in Bach. In the 1920s and 1930s, studies about this subject started popping up, seemingly out of nothing. This brings one to the inescapable conclusion that this branch of musicology has a direct relationship with modernity. Werker's publication in 1922 was revolutionary. Werker used a strictly analytical method and thereby refrained from the number symbolism in formal structure that would later gather such a following. His analysis was reviled by the establishment in his field, but got immediate recognition from Schönberg.[84] The composer and the theorist had the same leaning towards objectivity. Werker didn't say much about the spiritual background of his theory, but Schönberg made no secret of the fact that the esotericism of Emanuel Swedenborg (1688–1772), among other things, inspired his idea of the unity of musical space.[85]
In all the arts in the 1920s, spirituality, a Neoplatonic belief in an objective world of ideas, often expressed in esoteric or theosophical terms, played an important role. Typical for this time, in this connection, are the work and the ideas behind it of the painter Piet Mondriaan (1872–1944). Another typical example of this esoteric tendency is the formalism, founded in mysticism, of the architect Karel de Bazel (1869–1923), who was on good terms with the theosophist Mathieu Schoenmaekers (1875–1944).[86] A clear example of this esoteric tendency in music is the composer Joseph Matthias Hauer (1883–1959), who was highly esteemed by Schönberg. Hauer's twelve-tone system had a spiritual foundation.[87] In the Netherlands we have the example of the composer of *Proeven van Stijlkunst,* Jacob van Domselaer (1890–1960).
In the visual arts of their contemporaries we see the same kind of an often esoterically-inspired theoretical basis as we see in music.[88] What has come down to the next generations is mainly the audible and visible exterior of this art, while the theoretical background has become dated and is often reshaped into an easily digestible ideology, in the form of a more general anti-subjective music criticism. When one looks at it more broadly, the question presents itself whether in the numerical research of musical form, in this sub-branch of musicology, just like in all artistic, as well as political and social life, the spiritual core of modernism doesn't eventually peter out, broaden into a conventional and more bourgeois kind of neo-roman-

ticism. With the pioneer Werker there was absolutely no question yet of the comfortable ideas and free association that we find in some of the later researchers. Modernity announced itself but didn't really press ahead.[89] What remained in musicological research was the claim of objectivity, but even on that point the spirit of the times has changed so much that one can only wonder how much longer this will last. A lonely highpoint in musical number interpretation was reached by Piet Kee (1927) in the 1980s when he made the minor triad the symbol of the major triad.[90]

Fritz Feldmann rejected the thesis that it could be a case of the aftereffects of romanticism, for according to him thinking in symbolic numbers was not done in the nineteenth century. For everyone knows, he added, that the romantic-artistic sensibility abhors counting.[91] In its not very scholarly logic, the argument 'for everyone knows' sounds a lot like 'that can't be a coincidence'. At odds with this, in any case, is the fact that in the middle of the nineteenth century the golden ratio acquired its special meaning. And strangely enough, that golden ratio was taken up again by researchers like Kramer, Clement and Van Houten.[92]

In spite of all the inadequacies that typifies a lot of the research since the 1950s, Tatlow makes a fervent plea for historical musicology.[93] But the historical arguments that would have to support her own theory of proportional parallelism all turn out to be faulty, unfortunately. The results of her analytical research could, however, still stand without historical arguments, if it turned out they could be statistically underpinned. Until further notice she could just drop the symbolic interpretation of her research's end result. Arguments to practice a historical musicology are there in any case. There is no reason for gloom on that account. Musicologists aren't just interested in the number of bars, but also in the creative spirit, in the question of who people were then and who we are today.

Notes

1. See Thijs Kramer, Zahlenfiguren im Werk Johann Sebastian Bachs, p. 5. The word 'symbol' is used here according to common practice in musicology. Kramer points out that 'symbol' is not always the best word in connection with numbers in music. 'Es hat den Anschein, daß für alle möglichen, insbesondere "höheren" Beziehungen nur das Wort "Symbol" zur Verfügung steht, als ob Begriffe und Begriffsschattierungen wie Abbild, Abbildung, Abzeichen [...] usw. verboten wären.'
2. Andreas Werckmeister, Musicalische Paradoxal-Discourse, p. 93. 'Hierdurch wird die Dreyfaltigkeit sehr fein abgebildet, nach unsern Glaubens-Articule, und Symbolis.'
3. Andreas Werckmeister, Musicae mathematicae hodegus curiosus, p. 147. 'Kan wohl etwa ein Gleichnüß deutlicher vorgestellet werden, welches uns das Drey-Einige Göttliche Wesen gleichsam in einem Spiegel zeige, als dieses?'
4. Johannes Lippius, Synopsis musicae novae, 'De triade musica'. 'Trias Harmonica Simplex & Recta Radix vera est Unitrisona omnis Harmoniae perfectißime plenißimaeque quae dari in Mundo potest, Sonorum etiam mille & millies mille, qui omnes referri posse debent ad partes ejus in Unisono Simplici & Composito, magniistius Mysterii DIVINAE solurn adorandae UNITRINITATIS Imago & Umbra (an ulla luculentior esse poßit, nescio).'
5. Andreas Werckmeister, Musicalische Paradoxal-Discourse, p. 95. 'Diese Zahl 6. wird auch ein Numerus Mundanus, und thierische Zahl genennet, denn gleich wie dieselbe mit ihrer vorhergehenden Zahl 5. eine traurige Consonantiam, und zwar die kleineste Superparticularem machet, also ist die thierische Natur, die aller geringste und die kleineste vor GOttes Augen.'
6. Ibidem, p. 96. 'Als 1.2.3.4.5.6-8 in C c g c' e' g' c'' denn wenn diese Zahl 8. nicht dabey wäre, so könten wir keine Sextam minorem in dieser Ordnung haben.' See Gioseffo Zarlino, Le istitutioni harmoniche, p. 25. 'Delle Proprietà del numero Senario, & delle sue parti; & come in esse si ritroua ogni consonanza musicale.' See Johannes Lippius, Synopsis Musicae Novae, zonder paginanummer. De Principiis Cantilenae Harmonicae Cognoscendae. See Sethus Calvisius, Exercitatio Musica tertia, p. 59.
7. Rolf Dammann, Die Musiktheorie des Andreas Werckmeister, p. 236-237. '[...] daß dem Komponisten die Zahlen auch über diesen engeren Rahmen [der Intervalle] hinaus für den architektonische Aufbau der Musik wichtig werden.'
8. See Johann Andreas Herbst, Musica poëtica, p. 1. 'Theoretica in contemplando & speculando saltem consistit, a Θεωρέω contemplor: So da allein im anschauen und

tieffen nachsinnen bestehet, und nur die Rationes und Proportiones, Art und Weiß der Music betrachtet, aber zu keiner Ubung fürgenommen wird, daher kompt Musicus Theoricus, der nur allein die blosse Wissenschafft hat, oder auffs wenigst davon zu discurriren und zu reden weiß.'

9. Andreas Werckmeister, Musicae mathematicae hodegus curiosus, p. 98. 'Wer nun diese Lehre de proportionibus wohl verstehet, [...] wird auch unterscheiden können, welche von beyden Satzen oder Progressen am besten und natürlichsten: ob der Progreß aus der Sexta major: in octavam besser, oder ob ex sexta minori ad octavam natürlicher sey? also: h d - c' c, oder b d - e' c.'

10. Ibidem, p. 101. 'Je näher wir nun bey der Unität bleiben können, je natürlicher und besser die Harmonia ist.'

11. Arnold Schering, Wilhelm Werker: Studien über die Symmetrie im Bau der Fugen, p. 87. 'Werkers Dämon, der ihn verhext hat und schwerlich wieder loslassen wird, ist die Zahl, besser die Zahlensymbolik.'

12. See Wilhelm Werker, Die Matthäuspassion, p. 10. 'Aus Liebe zur Zahlensymmetrie hat der Gestalter Bach über den bibeltreuen Lutheraner gesiegt.'

13. Wilhelm Werker, Studien über die Symmetrie im Bau der Fugen und die motivische Zusammengehorigkeit der Präludien und Fugen des 'Wohltemperierten Klaviers' von Johann Sebastian Bach, p. 195. 'Stets handelt es sich um den betriebsamen Mathematiker, niemals um den müßigen oder gar mystischen Zahlenspekulanten!'

14. Philipp Spitta, Johann Sebastian Bach, band II, p. 712. Johann Nikolaus Forkel, Über Johann Sebastian Bachs Leben, Kunst und Kunstwerke, p. 21. 'Wenn nehmlich Jemand an der einen Ecke des langlicht viereckichten SaaIs oben ganz leise gegen die Wand einige Worte sprach, so konnte es ein Anderer, welcher übers Kreuz an der andern Ecke mit dem Gesichte gegen die Wand gerichtet stand, ganz deutlich hören, sonst aber Niemand im ganzen Saal, weder in der Mitte, noch an irgend einer andern Stelle. Diese Wirkung kam von der Richtung der an der Decke angebrachten Bogen, deren besondere Beschaffenheit er beym ersten Anblick entdeckte.'

15. Wilhelm Werker, Studien über die Symmetrie im Bau der Fugen und die motivische Zusammengehorigkeit der Praludien und Fugen des 'Wohltemperierten Klaviers' von Johann Sebastian Bach, p. 86. 'Es ist hohe Zeit, daß der Musiker Herz und Kopf wieder gemeinsam ausbilde, daß er sich schame, aufdringlich von seinen Gefühlen und Leidenschaften zu reden.'

16. Ibidem, p. 196. 'Wenn aber auch, wie das bei unserem Musikbetriebe wohl anzunehmen ist, diese erzieherischen Erfolge ausbleiben, hat diese Schrift doch ih-

ren Zweck erfüllt, wenn sie einem Bachfreunde es zur Gewißheit werden ließ, daß Johann Sebastian nicht nur ein warmes tiefes Herz, sondern auch kluge, weitblickende Kunstansichten hatte, daß er sich die von großen Schöpfern seiner Vorzeit ererbten Gestaltungs-, Wachstumsgesetze zu eigen machte und steigernd überbot.'

17. Ibidem, p. 1. 'Man betrachte die Musik der wenigen zum Schaffen gezwungenen Meister wie Naturprodukte.'

18. Bryan R. Simms, Who First Composed Twelve-Tone Music?, p. 133, note 27. 'Ich glaube wir arbeiten am selben Gegenstand: ich meine Sie [Matthias Hauer], er [Wilhelm Werker] und ich [Arnold Schönberg].' Leonard Stein, Schoenberg: Five Statements, p. 169. See Rudolf Stephan, Zum Thema 'Schönberg und Bach'.

19. Arnold Schering, Wilhelm Werker: Studien über die Symmetrie im Bau der Fugen, p. 73. 'Der Keim zu einem Musikstück entsprießend dem Boden abstrakten Denkens! Also Apotheose der musikalischen Impotenz! [...] Es verrät in dieser Hinsicht [die Psychologie] einen völlig uninteressierten Kopf.' See Rudolf Steglich, Wilhelm Werker – Die Matthauspassion, p. 79. 'Wilhelm Werkers Buch über Bachs Matthäuspassion ist (nicht weniger als das über das Wohltemperierte Klavier) in diesem Sinne [des Lebenszerfalls in diesem vergangenen Jahrzehnt] eine katastrophale Erscheinung in unserm Schrifttum über Musik.'

20. Wilhelm Werker, Studien über die Symmetrie im Bau der Fugen. Vorwort. 'Geboren wurde das Buch [...] aus der Gewißheit, daß der Weg der Moderne über Bach einer hohen, mathematisch wohlfundierten und phantasievollen Zukunftskunst entgegenführen muß.'

21. Ibidem, p. 294. 'Sogar die Söhne des Meisters scheinen keine Ahnung von dieser Architektonik in des Vaters Werken gehabt zu haben, sonst wären ihnen nicht ihre Formen in der Hand zerbrockelt.'

22. Ibidem, p. 348. 'Er wird in Bachischen Handschriften, etwa im Orgelbüchlein, die eigentümlich-geheimnisvollen algebraischen Formeln unter dem Choralvorspiele "Der Tag, der ist so freudenreich", die mit Generalbaßschrift und anderer Tabulatur nichts zu tun zu haben scheinen, nach einer anderen Seite zu deuten suchen.'

23. Ibidem, Vorwort. '[...] so daß ich zu dem Bekenntnis gezwungen bin, eigentlich außer dem Wohltemperierten kein Buch benützt zu haben.'

24. Wilibald Gurlitt, Zur biographie von Wilhelm Werker, p. 330. 'Wilhelm Werker gehört zu den ersten, die Zahlenbeziehungen als Mittel für die formale Gestaltung im Werk Joh. Seb. Bachs untersuchten. Diese Untersuchungen waren zu Lebzeiten ihres Verfassers lebhaft umstritten.'

25. Wolfgang Graeser, Bachs 'Kunst der Fuge', p. 12. 'Es ist ein beinahe aussichts-

loses oder zum mindestens vermessenes Unterfangen, mit den Mitteln unserer heutigen Musikwissenschaft an ein so enorm schwieriges Werk, wie die Kunst der Fuge, von der formalen Seite heranzutreten. Wir sind genötigt, statt mit einer wohldurchdachten und eindeutigen wissenschaftlichen Sprache und Terminologie, mit hinkenden Vergleichen, technischen Bezeichnungen aus anderen Gebieten und trügerischen Analogien zu arbeiten.' See Rudolf Wille, Musiktheorie und Mathematik, p. 4.

26. Wolfgang Graeser, Bachs 'Kunst der Fuge', p. 71. 'Die Gebilde, die die Natur aus uns und durch uns schafft, müssen sich also auch ihren großen Gesetzen fügen. Wir tuen in diesem Sinne nichts anderes als die Naturwissenschaften auch, nur daß unsere Methoden, denen der exakten Wissenschaften gegenüber noch ein Minimum an Zuverlässigkeit bieten.'

27. See Heinrich Schenker, Brief aan August Halm, Galtür 1927. 'Gegen Jöhde's, Werker's, Graeser's Bach Unternehmungen dürfte die kleine Abhandlung zunachst vielleicht wenig ausrichten, aber wenn es einem Bach so schlecht mit diesen Unmenschen geht, warum sollte ich es besser haben wollen? Noch vor 4-5 Jahren hatte ich einen solchen Bach-Verfall nicht anzunehmen gewagt, obwohl ich seit 30 Jahren auch i. meine Leser auf das Schlimmste gefaßt mache.'

28. Friedrich Smend, Luther und Bach, p. 19, 20. 'In der Matthäus-Passion finden wir 27 biblische Berichtsabschnitte, so wie Bach den Text durch die Betrachtungen gliedert. 3 x 3 x 3 deutet auf die hochheilige Trinität; Gottes Wort wird hier verkundigt. Und wenn wir die 3 x 3 x 3 auf den Menschen beziehen, so deutet dies Zahlen-Symbol auf den hochheiligen christlichen Glauben; denn nur im Glauben kann diese Verkündigung recht gehört werden. Machen wir uns die Mühe und zählen einmal die vollen Takte dieser 27 biblischen Berichtsstücke in der ganzen Matthäus-Passion zusammen, so kommen wir zu 729 (= 27 x 27) Takten.'

29. Friedrich Smend, Kirchen-Kantaten, Heft 3, p. 5 e.v. 'Hier wird mit Worten und Zahlen ein Spiel getrieben.'

30. Ruth Tatlow, Bach and the Riddle of the Number Alphabet, p. 30. 'Nowhere in his published works did Smend acknowledge Dieben's role in the evolution of the number-alphabet theory.'

31. Henk Dieben, Getallenmystiek bij Bach, slot, p. 48. Johann Gottfried Walther, Musicalisches Lexicon, p. 64. 'Dieser Remarque hat den Leipziger Hrn. Bach zum Erfinder.' The remark came from Johann Nicolaus Bach. See Hans T. David and Arthur Mendel (ed.), Christoph Wolff (rev.), The New Bach Reader, p. 295. Moreover Walther doesn't write about number values.

32. Ruth Tatlow, Bach and the Riddle of the Number Alphabet, p. 30. A letter

from Smend to Jansen contains a moral justification which reads in English translation: 'On the other hand, I too had something to give to the conversation, and did not need to take alone.'
33. Ibidem, p. 32. Smend swrites to Jansen in English translation: 'His presumptious tone is obnoxious (there's no other word for it!).'
34. Martin Jansen, Bachs Zahlensymbolik, p. 97. 'Der Zugang zu dem Problem "Bach und die Zahl", der durch den allzu kecken und unvorsichtigen Vorstoß W. Werkers verschüttet war, soll im folgenden wieder geöffnet werden.'
35. Ibidem, p. 96. 'Dem Altertum und Mittelalter war sie [die Zahl] daneben und darüber hinaus ein die Mannigfaltigkeit der geschaffenen Dinge ordnendes Prinzip und damit Ausdruck der den Kosmos regierenden Gesetze und Zeichen göttlichen Willens. [...] In der Musik ist diese Denk- und Arbeitsweise über das eigentliche Mittelalter hinaus gültig.'
36. George J. Buelow, 'Andreas Werckmeister', in The New Grove Dictionary of Music and Musicians. 'But in many of his views he remained a mystic and decidedly medieval.' See Arie Eikelboom, Jesu, meine Freude, p. 20. 'Werckmeister gaat terug op de muziekbeschouwing van de Middeleeuwen.'
37. Rolf Dammann, Zur Musiklehre des Andreas Werckmeister, p. 213. 'Diese im Zuge der reformatorischen Musikanschauung jüngst eingeschärfte und akzentuierte Haltung des Musikers wird von W. den nachwirkenden humanistischen Anschauungen sowohl als auch dem in seiner Zeit aufkommenden aufklärerischen Rationalismus gegenuber nachdrücklich betont.'
38. Andreas Werckmeister, Cribrum musicum, p. 4. 'Nun bestehet ja der Grammaticorum fundament auf der blossen Autorität, und Gewonheit der Autorum: Unsere Fundamenta Musica aber beruhen nicht alle in auf der Autorität, sondern haben auch guten Grund in der Natur.' See Ernst Lewalter , Spanisch-jesuitische und deutsch-lutherische Metaphysik des 17. Jahrhunderts, p. 9. 'Man hat die Scholastik von den Jesuiten rezipiert, um diese mit ihren eigenen Waffen schlagen zu konnen.' See Pieter Bakker, Harmonische Zahlen, p. 17 ff.
39. Hans Brandts Buys, De Passies van Johann Sebastian Bach, p. 159. 'Tijd moet andere eisen stellen dan ruimte, tijdelijke symmetrie moet anders zijn dan ruimtelijke, tijdelijke verhoudingen en projecties anders dan ruimtelijke.'
40. Ibidem, p. 176. 'Het ontbreken van symmetrie in het tweede deel en de aanwezigheid ervan in het eerste deel brachten mij er toe, de beide reeksen op elkaar te leggen en het centrum van het eerste deel te laten samenvallen met het centrum van het gehele werk. Het resultaat is het Kruis.'
41. Hans Brandts Buys, De passies van Johann Sebastian Bach, p.127. 'Daarmee is

gezegd dat de onderkenning van de symbolen de basis dient te zijn van elke interpretatie.' See Philipp Spitta, Johann Sebastian Bach, band II, p. 916. '[...] und soll wie aller Music, also auch des General Basses Finis und End Ursache anders nicht, als nur zu Gottes Ehre und Recreation des Gemüths seyn. Wo dieses nicht in Acht genommen wird da ists keine eigentliche Music sondern ein Teufliches Geplerr und Geleyer.' See Hans T. David and Arthur Mendel (rev. Christoph Wolff), The New Bach Reader, p. x. 'A very few documents of questionable value have been dropped, as has The Precepts and Principles ... for Playing a Thorough Bass.'

42. Hans Brandts Buys, De passies van Johann Sebastian Bach, p. 7, 11. 'Indien men er prijs op stelt deze meer intellectuele en zeker niet minder belangrijke zijde van de schoonheidsbelevenis te verdiepen, zal men zich enige moeite moeten getroosten.' 'Hoezeer het ook wenselijk zou zijn, daarom de oude instrumenten in ere te herstellen – het klankkarakter is toch wel een van de belangrijkste elementen der muziek – dit zal voorlopig wel een onvervulbare wens blijven. Bij het beoordelen van de oude muziek echter zijn we wel degelijk verplicht, ons rekenschap te geven van dit belangrijke feit.'

43. See Arnold Schering, Bach und das Symbol, p. 40. 'Fragt man nun, worin das Wesen der Vergeistigung der Ausdrucksmittel besteht, so kann darauf mit einem einzigen Worte geantwortet werden: in der Befähigung als Symbol zu wirken.'

44. Fritz Feldmann, Numerorum mysteria, p. 116. 'Aber zu dieser vom Quadrivium und seinem numerus her geläufigen Blickrichtung kommt im Zeitalter des Humanismus die des Trivium verstärkt hinzu.'

45. Ibidem, 117. 'Stets hilft hier die textbezogene Zahlensymbolik, die Übertretung des Redictae-Verbotes zurechtfertigen, darüber hinaus aber kann es als erwiesen gelten, daß Tinctoris kunstvolle Abweichungen von der Norm "sicut figurae rationabiles a grammaticis ornatus necessitatisve causa", also in bewußter Anlehnung an den sprachlichen Figurenbegriff, zuläßt. Gekoppelt mit den Figuren – oder, modern-unverbindlich formuliert, den Stilmerkmale –, an denen sich seit Dufay das musikgeschichtlich Neue zeigt, tritt auch die Zahlensymbolik deutlich in Erscheinung.' See Joannes Tinctoris, Tractatus de musica (red. Edmond de Coussemaker), Liber de arti contrapunti 1477, Lille 1875, p. 378.

46. Fritz Feldmann, Numerorum mysteria, p. 127. 'Nicht, daß sie [die Zahlensymbolik] häufig bei Händel angewandt sei, soll damit behauptet werden, sondern daß sie überhaupt noch bei ihm nachweisbar ist, daß sie dem großen Meister nicht fremd war.'

47. Walter Blankenburg, Einführung in Bachs h-moll-Messe, p. 80. 'Dieser dreigeteilte doxologische Abschluß des zweiten Artikels umfaßt 131 Takte. Ist es

Zufall?' Rolf Dammann, Der Musikbegriff im deutschen Barock, p. 87-88. 'Im Ostinato-Teil [Passacaglia c-moll] wird das 8 (= 2 x 4) Takte messende Thema 21 mal als Grundlage hörbar; 21 ist eine der Theologie bekannte und J.S. Bach geläufige Symbolzahl.'

48. Rolf Dammann, Zur Musiklehre des Andreas Werckmeister, p. 221, 236, 237. 'Der *Music-Bau,* als architektonisches Ordnungsgeflige und als Vorbild fur die Komposition beruht auf dem Fundament der Natur.' 'Aus der Kenntnis der *Würckungen* der Proportional-Zahlen in der Harmonia und in der rhythmisch-mensuralen Verlaufsgestalt der Musik wird es einleuchtend, daß dem Komponisten die Zahlen auch über diesen engeren Rahmen hinaus für den architektonische Aufbau der Musik wichtig werden.' Der Musikbegriff im deutschen Barock, p. 86, 91. 'Er [Werckmeister] mahnt, daß der *Music-Bau* in Stand gehalten und eine Komposition aufs (mathematische, naturliche) *Fundament gebauet* werden müsse.' 'Über die rationale Grundlage der Harmonik hinaus können die Zahlen fur den Werkaufbau maßgeblich werden.'

49. Vgl. Ruth Tatlow, Theoretical Hope, p. 55. 'He [Werckmeister] understood that every aspect of musical order strives after the equality of 1 : 1, including musical pitch, the degrees of the scale, the bar, speed, and range.'

50. Kees van Houten en Marinus Kasbergen, Bach en het getal, p. 11. 'Onze studie kan dan ook als een kritische voortzetting van het werk van Dieben, Ketting en Bronkhorst beschouwd worden.'

51. Ibidem, p. 59, 60. 'In het boek "Musicalische Paradoxal-Discourse" (1707) van Andreas Werckmeister worden de getallen van de boventonen 1, 2, 3, 4, 5, 6 en 8 "die wahren Radical-Proportiones der Harmoniae" genoemd.' Ook hier de verbinding met de Middeleeuwen. 'Middeleeuwse bouwmeesters schiepen met oneindige toewijding hun kathedralen als machtige en magische symbolen [...]. Bach's scheppingen onstonden, naar onze opvatting, vanuit een soortgelijke mentaliteit.' See Andreas Werckmeister, Musicalische Paradoxal-Discourse, p. 92.

52. Ibidem, p. 64. 'Met name de getallen van Bach's sterfdatum en/of sterfjaar blijken voortdurend te liggen op een zeer markante plaats in een compositie. In meer dan de helft van de gevallen betreft het dan het getal 372! De overtuigingskracht van al deze voorbeelden lijkt ons zo groot, dat men steeds meer tot het besef zal moeten komen, dat er van toeval geen sprake kan zijn.'

53. Wilhelm Werker, Studien im Symmetrie im Bau der Fugen, p. 193. 'Ist es ein Zufall, daß hier wie im Präludium [der es-moll-Fuge] wieder 29 Spiegelungen gezählt werden, 87 : 3 = 19!' Since the subject is chance and probability the exclamation mark should become a classic. See note 52.

54. Thijs Kramer, Zahlenfiguren im Werk Johann Sebastian Bachs, p. 54. 'Eine oft gestellte Frage ist: Hat man im Bachs Familien- und Bekanntenkreis etwas von Bachs numerischen Kompositionsweise gewußt?'
55. Wolfgang Wiemer, Carl Philipp Emanuel Bachs Fantasie in c-Moll, p. 174. 'Im ersten Satz also zweimal, wenn auch transponiert, ein B-A-C-H: in der Exposition plaziert auf Takt 14, in der Reprise auf Takt 41 – und das bei Carl Philipp Emanuel Bach.'
56. Lorenz Mizler, Musicalische Bibliothek, band IV, p. 158-176. The counting of the number of lines by Kramer is inaccurate.
57. Ibidem, p. 107. '14. Johann Sebastian Bach, Capellmeister und Musikdirector in Leipzig. Trat in die Gesellschafft im Jahr 1747 im Monat Junius.'
58. Thijs Kramer, Zahlenfiguren im Werk Johann Sebastian Bachs, p. 53. 'Die jeweiligen Enden der einzelnen Buchstaben laufen in einer punktförmigen Verdikkung aus. Da das B am unteren Ende mit zwei "Verdickungen" versehen ist, gibt es also insgesamt nicht 12, sondern 14 derartiger "Punkte".'
59. Heinrich Edelhoff, Johann Nikolaus Forkel, p. 44. 'Selbst der in der Mathematik so gelehrte Johann Sebastian Bach habe sich in diesen Fragen nach der Natur, nicht nach der Regel gerichtet, und die ganze Mathematisiererei habe noch nicht einmal den Erfolg gehabt, die Durchführung einer einwandfreien Temperatur zu gewahrleisten.'
60. Albert Clement, Der dritte Teil der Clavierübung von Johann Sebastian Bach, p. 329-330. 'Die Zahlen 2, 3, 5, 8, 13 (usw.) bilden die besagte Summenreihe mit der üblichen Teilung nach dem Goldenen Schnitt.' Thijs Kramer, Zahlenfiguren im Werk Johann Sebastian Bachs, p. 54. 'Die Verhältnisse der Arithmoi von Ιησους (888), Χριστος (1480) und deren Summe (2368) entsprechen der *sectio aurea:* 0,6 : 1 : 1,6 = 3 : 5 : 8.' See also note 92.
61. Albert van der Schoot, De ontstelling van Pythagoras, p. 179, 408. 'Contrary to common opinion, it is not until the nineteenth century that the golden section is hailed as an aesthetic ideal.'
62. Ibidem, p. 378. 'Als in de Renaissance in numerieke termen gedacht wordt over de muzikale ordening, dan wordt daarbij in de eerste plaats gedacht aan de *intervalverhoudingen.* Die zijn – althans in de theorie, dat wil zeggen in de gesanctioneerde stemmingen – altijd rationaal.'
63. Andreas Werckmeister, Musicalische Paradoxal-Discourse, p. 98. '[...] und Vitruvius beweiset daß auch die Schönheit der Architectur in den musicalischen Proportionibu bestehe.'
64. Albert Clement, Der dritte Teil der Clavierübung von Johann Sebastian Bach,

p. 66. 'Daß es jedoch in dieser Komposition, wie in BWV 669, genau sieben Engführungen gibt, dürfte kaum Zufall sein, gilt doch die Zahl sieben von alters her als numerus perfectus, eine Zahl also, die dem "Kyrie" durchaus angemessen ist.'
65. See Ruth Tatlow, Text, the Number Alphabet and NumericalOrdering in Bach's Church Cantatas, p. 127. 'Is this yet another chance correlation?'
66. Arie Eikelboom, Jesu meine Freude, p. 402. 'Verwijzingen naar getallen heb ik op grond van Bachs contemporaine theologische en muzikale bronnen kunnen duiden. Met name het werk van Andreas Werckmeister was daarbij verhelderend.'
67. Joseph Riepel, Anfangsgründe zur musicalischen Setzkunst, Antwortschreiben. Zweites Capitel: Grundregeln zur Tonordnung insgemein, Innhalt. 'Die mathematische Rationalrechnung hilft nichts zur Composition.'
68. Ruth Tatlow, When the Theorists Are Silent, p. 203. 'After a closer study of his [Smends] sources, however, it was clear not only that his interpretation was faulty, but that he had no historical or documentary evidence for his fundamen- tal premise, that Bach used numbers when he composed.'
69. Ruth Tatlow, Bach's Parallel Proportions and the Qualities of the Authentic Bachian Collection, p. 135. 'Results from the newly-formulated theory of proportional parallelism show three characteristics common to all Bach's published collections: 1. each collection has numerical proportions formed by the number of bars at two or more constructional levels, 2. the number of bars at the largest structural level is almost invariably a multiple of 10, and 3. either the total number of bars or the key patterns form a recognisable signature.' p. 148. 'Intervals in music were expressed in terms of perfection: the perfect unison 1 : 1, the perfect octave 1 : 2, and the perfect fifth 2 : 3. Was Bach seeking to attain perfection by creating perfect proportions at many structural levels of his collections?'
70. Ruth Tatlow, Text, the Number Alphabet and Numerical Ordering in Bach's Church Cantatas, p. 130. 'In the sample of 79 cantatas I have found a correlation between the text and numbers of bars in 52 different cantatas.'
71. Ruth Tatlow, Bach and the Riddle of the Number Alphabet, p. 128. 'No longer should analysts of Bach's music quote Smend's work as a reliable source.' Ruth Tatlow, Text, the Number Alphabet and Numerical Ordering in Bach's Church Cantatas, p. 130. 'And yet, contrary to my original thoughts, the occurences are still too frequent to write the experiment off as manipulation or chance occurence.'
72. Ruth Tatlow, Theoretical Hope, p. 43. In one paragraph the researcher uses both 'method' and 'theory' to describe her proportional parallelism.
73. Ruth Tatlow, When the Theorists Are Silent, p. 203. 'The priority is to establish historical plausebility that Bach used numbers as a tool when he composed.

Only then will it be clear which forms of enumeration, operation and translation he used, and only then will analysts deciphering his compositional process be able to make valid interpretations.'

74. Ruth Tatlow, Theoretical Hope, p. 49. 'Harmony has had many shades of meaning since it was coined as a concept in classical times.'

75. Heinz von Loesch, Der Werkbegriff in der protestantischen Musiktheorie des 16. und 17. Jahrhunderts: Ein Mißverständnis, p. 106, 115. 'Die Hauptthese der vorliegenden Arbeit besteht in der Annahme, daß die Begriffe "musica poetica" und "opus perfectum et absolutum" [...] kein Kompositions- und Werkbegriff in unserem Sinne sind.' 'Begann die Vorstellung überdauernder musikalischer Werke seit dem 19. Jahrhundert aber tatsächlich langsam selbstverständlich zu werden, so offenbar nicht, weil die "Musica poetica" ein Bewußtsein vom musikalischen Werk geschaffen hätte, sondern weil es das überdauernde Werk jetzt wirklich gab.'

76. See Pieter Bakker, Proportionen. This article delves further into the matter of Tatlow's source references.

77. Ruth Tatlow, Collections, bars and numbers, p. 37. Lorenz Mizler, Musicalische Bibliothek, deel IV, p. 158-176. 'Unser seel. Bach ließ sich zwar nicht in tiefe theoretische Betrachtungen der Musik ein, war aber desto stärcker in der Ausübung.' Johann Mattheson, Grundlage einer Ehrenpforte, p. 230-231. 'Dieser [Bach] hat ihm [Mizler] gewiß und wahrhafftig eben so wenig die vermeinten mathematischen Compositions-Gründe beigebracht, als der nachstgenannte [Mattheson].'

78. Johann Mattheson, Der vollkommene CapelImeister, p. 156, 157. 'Zwar darff niemand eben so scharff hierin verfahren, daß er Circkel und Maaß-Stab dabey zur Hand nähme.' Andreas Werckmeister, Musicalische Paradoxal-Discourse, p. 91-97. 'Diese Zahlen 1. 2. 3. 4. 5. 6. und 8. sind nun ein Corpus der völligen Harmonie.' See Wilhelm Werker, Studien über die Symmetrie im Bau der Fugen, p. 294. 'Johann Sebastian Bach nahm Zirkel und Maßstab, Schwert und Wage zur Hand.'

79. Bach-Dokumente, band I, p. 19. '[...] und sonst nach meinem geringen vermogen der fast auf allen Dorfschafften anwachsende kirchen *music,* und offt beßer, als allhier *fasonier*ten *harmonie* möglichst aufgeholffen hätte [...] .'

80. Michael Praetorius, Syntagma musicaum, p. 87-88. 'Denn weil ich nothwendig observiren müssen, wie viel tempora, wenn man einen rechten mittelmässigen Tact helt, in einer viertel Stunde musiciret werden konnen.'

81. Johann Buttstett, Ut mi sol re fa la, p. 169. 'So ist auch gewiß daß in dem Göttlichen Wesen, welches ab aeterno in aeternum, die höchste und vollkomneste, Harmonie ist; Folget dahero ex bona consequentia, daß die Harnonie auch ab

aeterno in aeternum sey. Und dieses wäre so dann mein stärckester Beweiß, daß die Music ewig bleiben werde.' Johann Mattheson, Das beschützte Orchestre, p. 474. 'Aber, daß ich darum glauben solte GOtt hätte keine andere, als die Aretinischen Sex voces im Vorrath, und hätte mit der Abmessung des Gnaden-Stuhls unsere Triadem in Musicis eben anzeigen wollen, so treuhertzig macht mich kein Mensch.'
82. See Pieter Bakker, Proportionen. 'Wenn Tatlow tatsächlich aus historischer Forschung musiktheoretischer, musikkritischer und biographischer Quellen zu diesem Ergebnis gelangt ist, kann nur von einem bemerkenswerter Fall von Serendipität die Rede sein. [...] Es wäre besser gewesen, wenn sie schlichtweg ihre symbolische Deutung fallen gelassen hätte und den Tatsachen ins Auge geblickt hätte, ohne voreilige Interpretationen heranzutragen.'
83. See Thijs Kramer, Zahlenfiguren im Werk Johann Sebastian Bachs, p. 54. 'Möglicherweise sind in Bachs Bibliothek mathematische Werke aus der Sammlung seines Vorgängers, des Thomaskantors Johann Kuhnau, eingegangen.' Ruth Tatlow, Bach and the Riddle of the Number Alphabet, p. 126. 'There can be little doubt that Walther passed on to his younger cousin [Bach] interesting ideas he received from the respected older composer and theorist [Werckmeister] .'
84. Leonard Stein, Schoenberg: Five Statements, p.161. 'I pointed out to him on this occasion that several things mentioned by Werker were not unfamiliar to me.' Rudolf Stephan, Zum Thema 'Schönberg und Bach', p. 241. 'Der bei Werker wichtige Gedanke, daß sich hinter den musikalischen Themen sinnstiftende Gestalten finden, mußte Schönberg zu jener Zeit, in welcher er die Methode der Komposition mit zwölf Tönen entwickelte, als eine höchst willkommene Bestätigung erscheinen. Und auch der Verweis auf zahlhafte Entsprechungen und hineingeheimniste Beziehungen waren ihm, der Ähnliches stets auch im Auge hatte, willkommen.'
85. Arnold Schoenberg, Stil und Gedanke, p. 115. *'Die Einheit des musikalischen Raumes erfordert eine absolute und einheitliche Wahrnehmung.* In diesem Raum gibt es wie in Swedenborgs Himmel (beschrieben in Balzacs *Seraphita)* kein absolutes Unten, kein Rechts oder Links, Vor- oder Rückwärts.' See Walther Klein, Das theosophische Element in Schönbergs Weltanschauung, p. 273. 'Schonberg ist Theosoph.'
86. Adriaan Wessel Reinink, K.P.C. de Bazel – Architect, p. 186, 188, 197. 'Hierop antwoordt De Bazel dat hij het globaal wel eens is met Pit, "alleen zou ik nog willen opmerken dat hoe ook de uiting zij ten allen tijde de lijn, het vlak bouwt en het vlak dus van structurale kracht suggestieve en werkzame uitdrukking zijn *moet* [cursivering van Reinink]. Aan deze eisch voldoet Renaissance en barok-kunst niet altijd,

het wordt dikwijls spel, los van die innerlijke doelwerking". De "lijn" had m.a.w. voor De Bazel zo'n normatieve geldigheid dat hij haar als grondwaarheid voor de gehele bouwkunst (en daarmee ook voor restauraties) stelde.' 'Merkwaardig, hoe De Bazels formalisme zich in "rationalistische" termen uit.' Reinink meant that apart from the resemblances of De Bazel and Mondriaan they differ as far as Mondriaan accepted the age of technics, and he therefore had a relation with the phenomen of his time, and De Bazel sticked to craftsmanship which was in the way of renewal. 'Toch hield hij open oog voor deze beweging en was geabonneerd op *De Stijl,* waarschijnlijk wegens zijn goede betrekkingen met Schoenmaekers.'

87. Monika Lichtenfeld, Josef Matthias Hauer, in The New Grove Distionary of Music and Musicians. 'He detested all art that expressed ideas, programmes or feelings, demanding a purely spiritual, supersensual music composed according to impersonal rules.'

88. See Marty Bax, Het web der schepping, p. 515. 'Mijn analyse wijst erop dat Mondriaan zijn kunsttheorie, waaraan hij zijn hele leven trouw is gebleven, heeft gebaseerd op centrale begrippen uit de moderne theosofie.'

89. See Jan Hendrik van den Berg, Gedane zaken, p. 248. 'Dat wil zeggen, de atonale componist verwerpt de klassiek-romantische traditie, met niet weinig nadruk, *en* hij aanvaardt die traditie, met niet minder nadruk. Hij wil wel en niet.'

90. Piet Kee, De geheimen van Bachs Passacaglia, p. 325. 'Werckmeisters tekst zorgt voor de verklaring [van het optreden van de gebroken drieklanken]: "ein Grund worauf die ganze Harmonia gebauet wird (...) denn 4.5.6. giebet Triadem Harmonicam." Dit is exact wat Bach hier in de praktijk brengt. Part 10 begint met de vierde boventoon c' en daarop wordt dan de drieklank gebouwd: 4.5.6. Niet in de "Sonis" c, e, g, maar in kleine terts.'

91. Fritz Feldmann, Numerorum mysteria, p. 102. 'Dabei soll der Frage, ob erst "in neuerer Zeit die Zahlensymbolik überschätzt", ja eine "Nachwirkung des romantischen Denkens des 19. Jahrhunderts" sei, nachgegangen werden; ist doch diese Behauptung erst dann beachtlich, wenn jenes symbolische Zahlen- denken bis etwa 1750 nicht wesentlich feststellbar wäre, dagegen im 19. Jahrhundert ernsthaft nachgewiesen werden könnte. Das letztere wird kaum gelingen, weiß doch jeder, daß der Abscheu vor der Arbeit des Zählens gerade in der romantischen Schaffensauffassung kulminiert.'

92. Kees van Houten, De Universele Bach voor kenner en liefhebber, p. 176. 'Het stuk [openingskoor van de Matthäus-Passion] telt 90 maten. De gulden snede 90 geeft een verdeling van 55,62 en 34,38. De meest belangrijke caesuur van het werk ligt precies na 56 maten, waar de retorische Confutatio begint met een nieuwe mu-

zikale thematiek (staccato-noten). Hierdoor ontstaat een structuurverdeling 56 - 34 [benadering van gulden snedeverhouding volgens reeks van Fibonacci].' See Piet Kee, De geheimen van Bachs Passacaglia, 317. 'Door het breekpunt ontstaat tevens de Gulden Snede, de divina proportio, de "goddelijke verhouding".' See Marty Bax, Het web der schepping, p. 518. 'Zeer invloedrijk [met betrekking tot de architectuurstijl van de Amsterdamse School], zij het op afstand, was de in Duitsland verblijvende Lauweriks. Hij ontwikkelde daar een totaal nieuwe theorie omtrent systeem en vormgeving, gebaseerd op de wiskundige reeks van Fibonacci en op het theosofische begrip Kundalini (slangenkracht), dat zich uit in meanders van allerlei soort.' See also note 53.

93. Ruth Tatlow, When the Theorists Are Silent, p. 203. 'The trend for developing interpretative and cognitive models where music is studied as a sounding rather than a written phenomenon has reduced musicology's traditional emphasis on research into the historical primary source. In this paper I will describe an exercise in experimental music theory based on primary sources, which resulted in the formulation of a new, historically informed theory.' The rest of her argument shows that Tatlow regards Mattheson's Der vollkommene Capellmeister as a primary source in connection with her research into Bach's structures.

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